A Summary of the Geodesic-Coherence Refinement for Fourier-Domain Photon-Ring Inference

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This note summarizes the arc across
A Fourier-Domain Analysis of BHEX for Photon-Ring Inference and
Geodesic-Provenance Coherence Bounds for Fourier-Domain Photon-Ring Inference, and is not affiliated
with the BHEX collaboration or mission team.

The original Fourier-domain note [1] began with a simple model for long-baseline black-hole interferometry,

$$y = \alpha g + p + \varepsilon,$$

where g is a photon-ring template, p is structured background emission, and ε is noise. Its main conceptual point was that **readability** and **recoverability** are not the same thing: a direct amplitude heuristic can lose clarity once the modulus nonlinearity distorts oscillation spacing, even though a structured inverse problem can still recover the ring stably. The mismatch between those two regimes is controlled by a ring-background coherence term together with nuisance size and noise.

The later geodesic-coherence note [2] asked what that coherence term really means physically. Its answer was to lift the nuisance model from visibility space back to photon initial-condition space and show that the relevant overlap can itself be bounded by the geometry of escaping geodesic families. This summary note records that arc in one place, so that a reader who has already seen the two notes can latch onto the intuitive flow and the final result.

1 The original Fourier-domain story

In the first note [1], the visibility data are modeled in a complex Hilbert space as

$$y = \alpha g + p + \varepsilon.$$

A simple ring-oriented heuristic reads off information directly from the visibility amplitude, while a plasma-aware estimator explicitly projects away or jointly models structured nuisance. In the simplest subspace version, the key overlap quantity is

$$\mu(g, \mathcal{P}) = \frac{\|\Pi_{\mathcal{P}} g\|}{\|g\|},$$

where $\Pi_{\mathcal{P}}$ is orthogonal projection onto a nuisance subspace $\mathcal{P}$.

The first lemma in that note made the mismatch story concrete: the discrepancy between a direct-readout estimator and a plasma-aware estimator is controlled by a coherence term of the schematic form

$$\mu(g, \mathcal{P}) \frac{\|p\|}{\|g\|},$$

together with noise contributions. So the heuristic does not fail arbitrarily. Its bias grows when the assumed ring template has significant projection into the nuisance family, or when the nuisance and noise become large relative to the ring.

The accompanying proposition then sharpened the conceptual message. Because the modulus is nonlinear, moderate contamination can shift amplitude extrema and distort oscillation spacing, so *clean readability* can break down even while a structured inverse problem remains locally identifiable and stable. In other words, **the heuristic readability limit can occur before the recoverability limit**. This was the first note's main structural contribution: it recast the BHEX long-baseline picture as a source-separation problem in which visual cleanliness is not the same thing as mathematical recoverability [3, 4].

2 Why provenance had to enter

That first framework was intentionally abstract. It treated the nuisance class geometrically, but did not ask where the nuisance actually comes from. The next step was to notice that photons are not arbitrary signals in visibility space. They arise from initial conditions

$$z = (p, \xi),$$

where p is an emission event and ξ is an initial null direction, and then propagate along null geodesics.

Let E denote the set of escaping photon initial conditions. The crucial refinement is that the observable set itself naturally splits as

$$E = E_{\mathrm{ring}} \dot{\cup} E_{\mathrm{bg}},$$

where E_{ring} consists of near-critical escaping geodesics that generate the photon-ring signal, while E_{bg} consists of ordinary escaping geodesics that generate the broader background. Captured photons do not contribute to the data, but the deeper point is that even within the observable sector, ring and background come from distinct provenance classes.

This reinterprets the Fourier-domain decomposition. Instead of treating p or q as an arbitrary structured contaminant, one writes the ring and the background as visibility-space images of transport operators acting on initial-condition densities. If

$$A_r = \mathcal{F} T_{E_{\mathrm{ring}}}, \qquad A_b = \mathcal{F} T_{E_{\mathrm{bg}}},$$

then one has

$$g = A_r f_\theta, \qquad q = A_b h,$$

for suitable emission densities f_θ and h. The nuisance family is therefore no longer generic; it is constrained to the provenance class

$$Q_{\mathrm{prov}} = \{A_b h : h \in \mathcal{A}\},$$

for an admissible emission class $\mathcal{A}$.

3 The main new theorem

The real mathematical advance of the second note [2] was not just the observation that Q_{prov} is smaller than an abstract nuisance class. It was an explicit upper bound on the provenance-constrained coherence itself. For a ring template g, define

$$\mu(g, Q_{\text{prov}}) := \sup_{q \in Q_{\text{prov}} \setminus \{0\}} \frac{|\langle g, q \rangle|}{\|g\| \, \|q\|}.$$

Assume the transport operators satisfy lower nondegeneracy bounds

$$\|A_r f\| \geq \sigma_r \|f\|, \qquad \|A_b h\| \geq \sigma_b \|h\|,$$

on the relevant classes.

If the operators admit a visibility kernel $K(u, z)$, define the cross-Gram kernel

$$G(z, z') := \langle K(\cdot, z), K(\cdot, z') \rangle_{\mathcal{H}}.$$

Then the key theorem says that

$$\mu(g, Q_{\text{prov}}) \leq \frac{1}{\sigma_r \sigma_b} \left(\int_{E_{\text{ring}}} \int_{E_{\text{bg}}} |G(z, z')|^2 \, d\nu(z) \, d\nu(z') \right)^{1/2}. \tag{1}$$

This is the centerpiece. The abstract overlap term from the original note is now lifted to photon initial-condition space and bounded by a cross-correlation integral over escaping geodesic families. That is what turns the provenance idea into a genuinely new theorem rather than a structural remark.

4 The geometric separation corollary

To turn the preceding theorem into a more geometric statement, the second note introduces a scalar criticality index $\chi(z)$ measuring how “ring-like” an escaping trajectory is. The specific choice can vary; it might be tied to winding depth, near-photon-region residence time, or another surrogate for criticality.

Assume there is a gap $\Delta > 0$ such that ring-generating and background-generating trajectories are separated in this index:

$$|\chi(z) - \chi(z')| \geq \Delta \quad \text{for } z \in E_{\text{ring}}, \ z' \in E_{\text{bg}}.$$

Assume also that the cross-Gram kernel decays exponentially with this separation:

$$|G(z, z')| \leq M e^{-\beta |\chi(z) - \chi(z')|}.$$

Then the coherence itself obeys the explicit bound

$$\begin{aligned} \mu(g, Q_{\text{prov}}) \leq \frac{M}{\sigma_r \sigma_b} & e^{-\beta \Delta} \\ & \cdot \sqrt{\nu(E_{\text{ring}}) \, \nu(E_{\text{bg}})}. \end{aligned} \tag{2}$$

This is the genuinely geometric conclusion. It says that the key overlap term in the Fourier-domain mismatch story becomes quantitatively small when the near-critical escaping flow and the ordinary escaping flow are dynamically well separated. The stronger statement is not merely that the nuisance class is smaller, but that its coherence with the ring can decay exponentially under a phase-space separation hypothesis.

5 What changed across the two notes

Taken together, the two notes tell a coherent story.

The first note [1] established that direct amplitude readability can fail before structured recoverability, with the mismatch controlled by coherence, nuisance magnitude, and noise. The second note [2] identified a physically meaningful way to sharpen that coherence term. By lifting the model to photon initial-condition space and splitting the escaping set into ring and background provenance classes, it proved that the relevant overlap is bounded by a cross-Gram integral over escaping geodesic families. Under a criticality-gap assumption, that overlap decays exponentially.

So the original question was:

Can the ring remain recoverable after it stops being cleanly readable?

The refined answer is:

> Yes, and the degree of recoverability is controlled by
>
> the geometric separation of escaping photon families.

That is the intuitive arc. The original note gave an abstract structural theory of mismatch and robustness. The second note showed that the key overlap term in that theory is not merely formal: under natural operator and separation assumptions, it is governed by null-geodesic phase-space geometry.

6 Relation to BHEX and computational outlook

This summary still sits squarely in the spirit of the BHEX long-baseline program [3, 4, 5]. BHEX emphasizes that sufficiently long baselines should reveal a more universal photon-ring signal beneath broader astrophysical emission. The first note recast that intuition as a Fourier-domain structured source-separation problem. The second note then identified a route by which the key overlap term in that source-separation problem could, in principle, be estimated from geodesic geometry.

The framework remains abstract until one chooses an admissible emission class, a concrete criticality index, and a practical way to estimate the cross-Gram kernel from ray tracing or simulation. But this is exactly the bridge the two notes were moving toward: from abstract mismatch and recoverability statements to physically informed BHEX-style inference.

7 Key interesting points

- The first note introduced a Fourier-domain structured source-separation framework and showed that readability can fail before recoverability [1].
- The second note lifted the nuisance model to photon initial-condition space and split the observable sector into ring-generating and background-generating escaping geodesics [2].

- The main new theorem bounded provenance-constrained coherence by a cross-Gram integral over escaping geodesic families.
- Under a criticality-gap assumption, that coherence decays exponentially with phase-space separation.
- The central advance is therefore not just “less nuisance,” but a concrete mechanism by which black-hole geodesic geometry sharpens the original Fourier-domain mismatch story.

References

[1] Jonathan R. Landers, *A Fourier-Domain Analysis of BHEX for Photon-Ring Inference*, 2026.

[2] Jonathan R. Landers, *Geodesic-Provenance Coherence Bounds for Fourier-Domain Photon-Ring Inference*, 2026.

[3] Black Hole Explorer Collaboration, *Black Hole Explorer: Motivation and Vision*, arXiv:2406.12917. `https://arxiv.org/abs/2406.12917`

[4] Black Hole Explorer Collaboration, *The Black Hole Explorer: Photon Ring Science, Detection and Shape Measurement*, arXiv:2406.09498. `https://arxiv.org/abs/2406.09498`

[5] Black Hole Explorer Collaboration, *Interferometric Inference of Black Hole Spin from Photon Ring Size and Brightness*, arXiv:2509.23628. `https://arxiv.org/abs/2509.23628`