Residual-of-Residual Geometry of NFW Fits to SPARC Rotation Curves

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Abstract

This note applies a residual-of-residual diagnostic to public SPARC galaxy rotation curves. The diagnostic follows the geometric-residual framework of Landers [1] and the synthetic NFW proof-of-concept in Landers [2]. For each usable SPARC galaxy, the baryon-subtracted acceleration residual is computed from the observed rotation curve and fixed stellar mass-to-light ratios, then fit with a pure Navarro-Frenk-White (NFW) halo and a cored/isothermal control profile. The object of interest is not only the fit score, but the leftover field $\epsilon_g(R) = \Delta g_{\mathrm{obs}}(R) - \Delta g_{\mathrm{candidate}}(R)$. Across 165 fitted galaxies, cored/isothermal residuals beat full-range NFW fits in 78.8 percent of cases. In a primary-quality sample of 131 galaxies, the fraction is 80.2 percent, and outer-region NFW fits leave a structured negative inner residual in 51.9 percent of cases. The result is not a falsification of cold dark matter. It is evidence that residual-of-residual fields are measurable on real data and can classify where simple source geometries fail.

1. Context

The parent geometric-residual paper [1] recasts the missing-mass problem as a comparison between the geometry inferred from observations and the geometry predicted from baryons alone:

$$\Delta G_{\mu\nu} := G_{\mu\nu}[g^{\mathrm{obs}}] - G_{\mu\nu}[g^{\mathrm{bar}}], \qquad T^{\mathrm{miss}}_{\mu\nu} := \frac{c^4}{8\pi G}\Delta G_{\mu\nu}. \tag{1}$$

If ordinary general relativity is retained, a dark-matter model is a candidate stress-energy source for this residual:

$$\Delta G_{\mu\nu} \overset{?}{=} \frac{8\pi G}{c^4} T^{\mathrm{candidate}}_{\mu\nu}. \tag{2}$$

In weak-field disk galaxies, rotation curves mainly constrain the Newtonian potential Φ:

$$v^2(R) = R\frac{\mathrm{d}\Phi}{\mathrm{d}R}. \tag{3}$$

The practical residual is therefore

$$\boxed{\Delta g(R) = \frac{V^2_{\mathrm{obs}}(R) - V^2_{\mathrm{bar}}(R)}{R}.} \tag{4}$$

The synthetic NFW note [2] showed that a pure NFW profile can match an outer flat-curve residual, but then tends to overpredict the central residual in a controlled toy target. This paper asks whether the same diagnostic can be applied to real astrophysical data.

2. Data and Residual Construction

The data are the public SPARC mass models of Lelli et al. [4], consisting of 175 disk galaxies with Spitzer photometry and accurate rotation curves. The mass-model table gives, at each radius, V_{obs}, its random uncertainty, and the gas, disk, and bulge velocity contributions. The sample metadata provide galaxy quality flags, inclinations, disk scale lengths, and flat-speed estimates.

The baryonic contribution is computed using fixed fiducial stellar mass-to-light ratios:

$$\Upsilon_{\mathrm{disk}} = 0.5, \qquad \Upsilon_{\mathrm{bulge}} = 0.7. \tag{5}$$

Since the SPARC gas velocity contribution may be signed, the baryonic speed term is

$$V^2_{\mathrm{bar}} = \mathrm{sign}(V_{\mathrm{gas}})V^2_{\mathrm{gas}} + \Upsilon_{\mathrm{disk}}V^2_{\mathrm{disk}} + \Upsilon_{\mathrm{bulge}}V^2_{\mathrm{bul}}. \tag{6}$$

The observed residual acceleration is then

$$\Delta g_{\rm obs}(R) = \frac{V_{\rm obs}^2(R) - V_{\rm bar}^2(R)}{R}. \tag{7}$$

The propagated acceleration uncertainty is approximated as

$$\sigma_{\Delta g}^2 = \left[\frac{2|V_{\rm obs}|\sigma_V}{R}\right]^2 + \left[0.05\frac{V_{\rm obs}^2}{R}\right]^2, \tag{8}$$

where the second term is a modest systematic floor. This is not a full SPARC halo-inference likelihood; it is a controlled residual-space diagnostic.

3. Candidate Residual Families

The NFW density profile [3] is

$$\rho_{\rm NFW}(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}. \tag{9}$$

With $x = r/r_s$,

$$M_{\rm NFW}(<r) = 4\pi\rho_s r_s^3\left[\ln(1+x) - \frac{x}{1+x}\right], \tag{10}$$

so the residual acceleration is

$$\Delta g_{\rm NFW}(R) = \frac{GM_{\rm NFW}(<R)}{R^2}. \tag{11}$$

The cored/isothermal control profile is

$$\Delta g_{\rm iso}(R) = \frac{v_0^2 R}{R^2 + r_c^2}. \tag{12}$$

It is included because it naturally approaches the flat-curve residual shape $\Delta g \propto 1/R$ outside its core. A logarithmic-tail comparison $\Delta g_{\rm log} = A/(R + R_0)$ is also fit, but the main physical comparison here is NFW versus the cored/isothermal control.

The diagnostic field is

$$\boxed{\epsilon_g(R) = \Delta g_{\rm obs}(R) - \Delta g_{\rm candidate}(R).} \tag{13}$$

For the outer-NFW test, NFW is fit only outside a galaxy-specific break radius, chosen from $2.2R_{\rm disk}$ when available and clipped to the central percentile range of the sampled radii. The question is then whether the NFW halo required by the outer residual overpredicts the inner residual.

A spherical effective-density proxy is also computed for diagnostics:

$$\epsilon_\rho(R) = \frac{1}{4\pi G R^2}\frac{\mathrm{d}}{\mathrm{d}R}\left[R^2\epsilon_g(R)\right]. \tag{14}$$

This is not a disk mass inversion. It is a standardized way to compare the sign and concentration of leftover source structure.

4. Sample and Outputs

The pipeline parsed 3,391 rotation-curve points for all 175 SPARC galaxies. Of these, 165 galaxies had enough valid points for the fitting pipeline. The primary-quality sample contains galaxies with SPARC quality flag 1 or 2, at least 8 rotation-curve points, and inclination above 30 degrees when available. This gives 131 galaxies. A stricter primary-flat subset requires measured $V_{\rm flat} > 0$, leaving 113 galaxies.

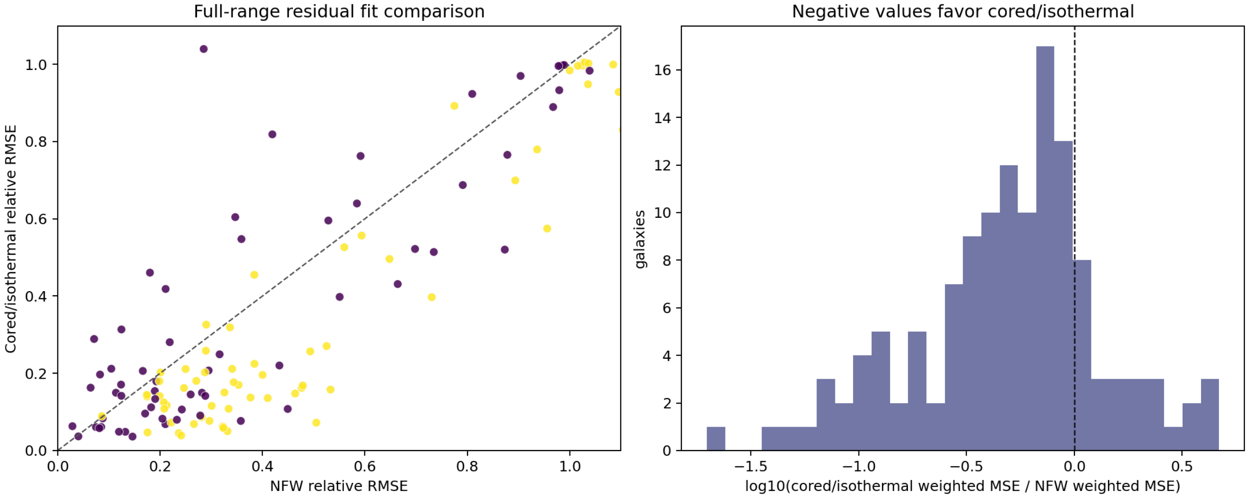
The experiment writes one row per galaxy, pointwise residual profiles, a population summary, source metadata, and figures. All files are generated by `run_sparc_nfw_residuals.py`.

5. Results

The first result is a direct fit comparison. In the full fitted sample, cored/isothermal residuals beat full-range NFW fits in 78.8 percent of galaxies. The median NFW relative RMSE is 0.337, while the median cored/isothermal relative RMSE is 0.203. In the primary-quality sample, cored/isothermal residuals beat NFW in 80.2 percent of galaxies, with median relative RMSE 0.206 versus 0.334 for NFW.

Sample	Galaxies	NFW RMSE	Cored RMSE	Cored beats NFW	Inner overshoot	Inner gap
All fitted	165	0.337	0.203	0.788	0.521	-0.449
Primary quality	131	0.334	0.206	0.802	0.519	-0.469
Primary flat	113	0.334	0.211	0.805	0.504	-0.449

Table 1: Population summary. NFW RMSE and cored RMSE are median full-range relative RMSE values. “Cored beats NFW” is the fraction of galaxies where the cored/isothermal weighted MSE is lower than the NFW weighted MSE. “Inner overshoot” is the fraction where the outer-NFW fit leaves a structured negative inner residual. “Inner gap” is the median normalized inner ϵ_g after outer-NFW fitting.


Full-range residual fit comparison

Negative values favor cored/isothermal

Figure 1: Population comparison of full-range residual fits. Left: each point compares the NFW and cored/isothermal relative RMSE for one galaxy in the primary-quality sample. Points below the dashed line favor the cored control. Right: the distribution of weighted-MSE ratios is shifted negative, meaning that cored/isothermal residuals beat NFW for most of the sample.

The second result is the residual-of-residual diagnostic. When NFW is fit to the outer region and then evaluated in the inner region, the primary-quality sample has a median normalized inner gap of -0.469. Under the current classification rule, 51.9 percent of primary-quality galaxies show central overshoot: the outer-matched NFW halo leaves a structured negative inner ϵ_g. In the primary-flat subset, the overshoot fraction is 50.4 percent.

The third result is the outer-slope behavior. Full-range NFW fits often reduce central overshoot by choosing a shape that is too shallow in the outer region. The present diagnostic labels 74.8 percent of primary-quality full-range NFW fits as shallow in the outer region. This is the real-data counterpart of the synthetic result: NFW can mimic the flat-curve residual over a scale-radius band, but fitting both the inner transition and outer tail with one pure NFW shape is frequently difficult.

6. Relation to Existing SPARC Halo Fits

This result is consistent with the broader SPARC halo-fit literature. Li et al. [5] fit all 175 SPARC galaxies with several dark-matter halo families, including pISO, Burkert, NFW, Einasto, DC14, coreNFW, and Lucky13. They report that cored halo models such as DC14 and Burkert generally fit the rotation curves better than cuspy NFW. The present work is not a replacement for that Bayesian halo catalog. It is a different projection of the same problem: instead of only ranking halo profiles, it explicitly records the leftover acceleration field $\epsilon_g(R)$ and asks whether the leftover has a repeatable geometry.

The answer is yes. The leftover is not merely a larger chi-square. In many galaxies it is a coherent inner negative residual when NFW is forced to match the outer region. This converts a familiar core/cusp tension [7] into an object that can be classified across galaxies.

7. Limitations

Several caveats are important. The stellar mass-to-light ratios are fixed, not marginalized. Distance and inclination uncertainties are not varied. No cosmological mass-concentration prior is imposed. The NFW fit

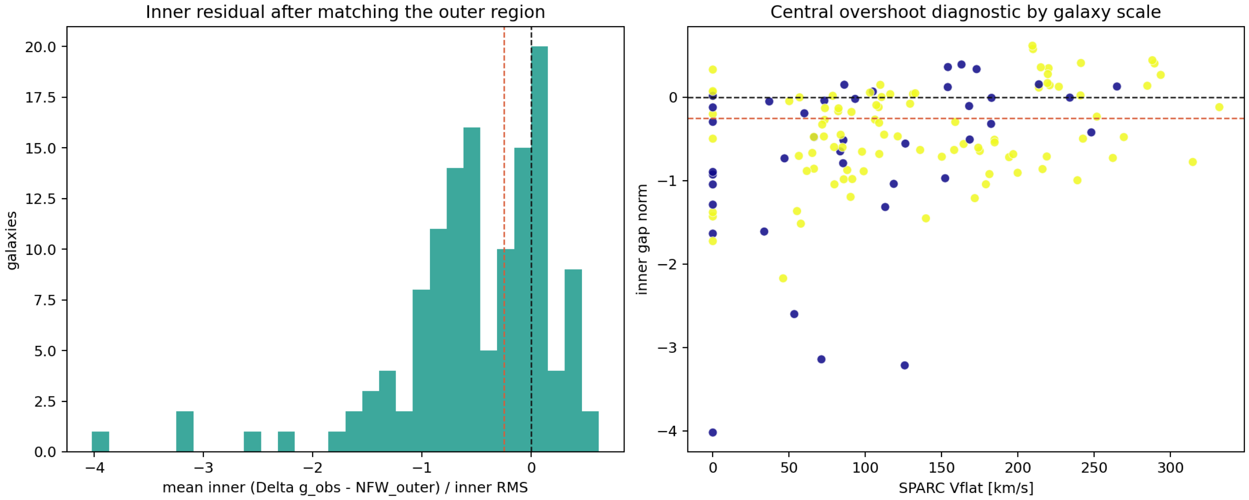


Figure 2: Residual-of-residual behavior after matching NFW to the outer region. The left panel shows the distribution of normalized inner $\Delta g_{\mathrm{obs}} - \Delta g_{\mathrm{NFW,outer}}$. The median is negative in every reported sample. The right panel shows the same diagnostic against SPARC V_{flat}, with point color indicating the SPARC quality flag.

is a simple two-parameter residual fit, not a full ΛCDM halo inference. The cored/isothermal profile is a control shape, not a claimed fundamental theory. The density proxy is spherical and diagnostic, while SPARC galaxies are disks. The lensing-sensitive metric potential $\Phi + \Psi$ is not constrained by these rotation data.

Therefore the claim is methodological and empirical, not a final physical verdict. The result does not falsify cold dark matter. It shows that residual-of-residual geometry can be measured on public data and that simple NFW residuals leave structured, classifiable leftover fields in a large fraction of SPARC galaxies.

8. Conclusion

The synthetic NFW note [2] predicted a clear failure mode: pure NFW can match an outer flat-curve residual or reduce central overshoot, but it does not naturally do both in a toy target. The SPARC experiment finds the same pattern in real data as a population tendency. Cored/isothermal residuals beat full-range NFW fits for about 80 percent of the primary-quality sample, and outer-NFW fits leave a structured negative inner residual in about half of the sample.

The main contribution is the object

$$\boxed{\epsilon_g(R) = \Delta g_{\mathrm{obs}}(R) - \Delta g_{\mathrm{candidate}}(R).} \tag{15}$$

It turns model mismatch into a geometric field. Once that field is measured galaxy by galaxy, the next question is no longer just which named halo profile has the lowest fit score. The sharper question is what physical mechanisms produce the observed families of leftover residual geometry.

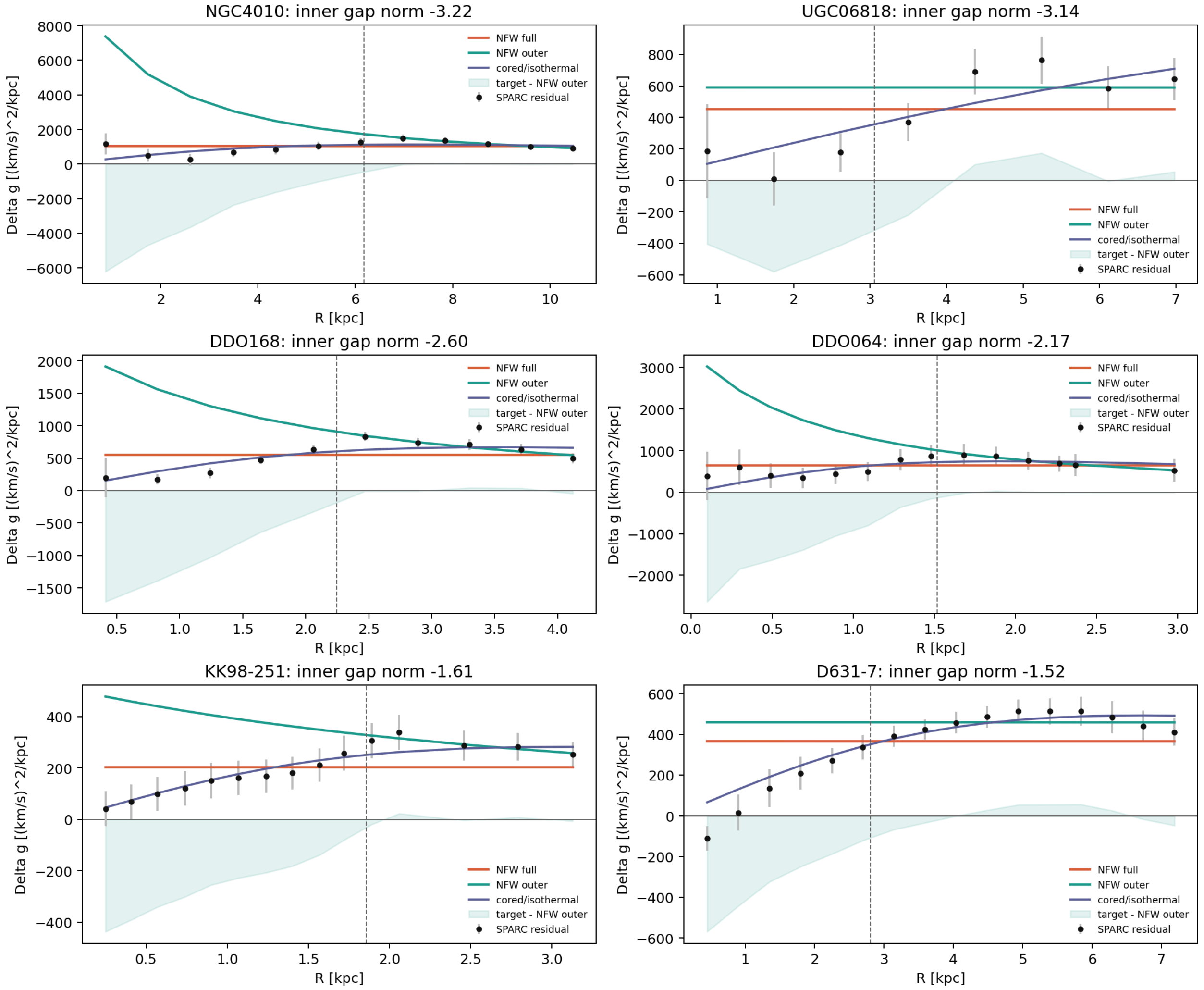


Figure 3: Example galaxies with strong negative inner residuals after the NFW outer fit. The filled region is $\Delta g_{\mathrm{obs}} - \Delta g_{\mathrm{NFW,outer}}$. The dashed vertical line marks the inner/outer split used for the outer-NFW fit. These panels show the residual-of-residual field directly rather than only reporting a scalar fit score.

References

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